FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences
Physics
Grades 10 - 12

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Contents

Chapter 24

Doppler Effect - Grade 12

24.1 Introduction

Have you noticed how the pitch of a car hooter changes as the car passes by or how the pitch of a radio box on the pavement changes as you drive by? This effect is known as the **Doppler Effect** and will be studied in this chapter.

Interesting Fact

The Doppler Effect is named after Johann Christian Andreas Doppler (29 November 1803 - 17 March 1853), an Austrian mathematician and physicist who first explained the phenomenon in 1842.

24.2 The Doppler Effect with Sound and Ultrasound

As seen in the introduction, there are two situations which lead to the Doppler Effect:

1. When the source moves relative to the observer, for example the pitch of a car hooter as it passes by.
2. When the observer moves relative to the source, for example the pitch of a radio on the pavement as you drive by.

Definition: Doppler Effect
The Doppler effect is the apparent change in frequency and wavelength of a wave when the observer and the source of the wave move relative to each other.

We experience the Doppler effect quite often in our lives, without realising that it is science taking place. The changing sound of a taxi hooter or ambulance as it drives past are examples of this as you have seen in the introduction.
The question is how does the Doppler effect take place. Let us consider a source of sound waves with a constant frequency and amplitude. The sound waves can be drawn as concentric circles where each circle represents another wavefront, like in figure 24.1 below.

The sound source is the dot in the middle and is stationary. For the Doppler effect to take place, the source must be moving. Let's consider the following situation: The source (dot) emits one peak (represented by a circle) that moves away from the source at the same rate in all directions.

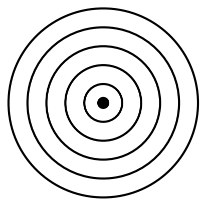
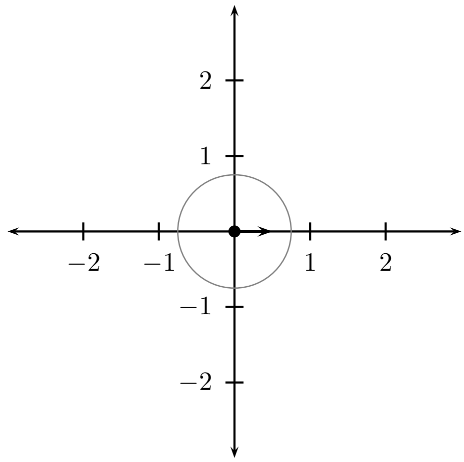
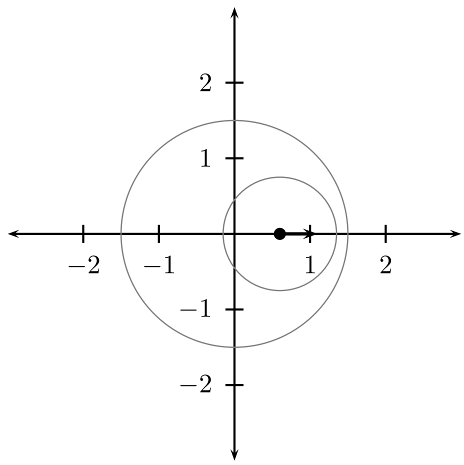


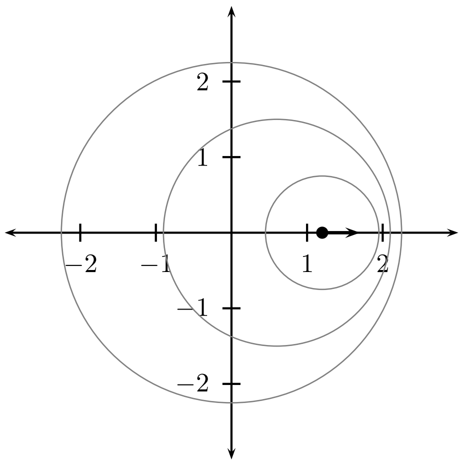
Figure 24.1: Stationary sound source



As this peak moves away, the source also moves and then emits the second peak. Now the two circles are not concentric any more, but on the one side they are closer together and on the other side they are further apart. This is shown in the next diagram.



If the source continues moving at the same speed in the same direction (i.e. with the same velocity which you will learn more about later). then the distance between peaks on the right of the source is the constant. The distance between peaks on the left is also constant but they are different on the left and right.

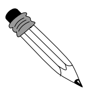


This means that the time between peaks on the right is less so the frequency is higher. It is higher than on the left and higher than if the source were not moving at all.

On the left hand side the peaks are further apart than on the right and further apart than if the source were at rest - this means the frequency is lower.
When a car appoaches you, the sound waves that reach you have a shorter wavelength and a higher frequency. You hear a higher sound. When the car moves away from you, the sound waves that reach you have a longer wavelength and lower frequency. You hear a lower sound. This change in frequency can be calculated by using:

$$f_L = \frac{v \pm v_L}{v \mp v_S} f_S \qquad (24.1)$$

where f_L is the frequency perceived by the listener,
f_S is the frequency of the source,
v is the speed of the waves,
v_L the speed of the listener and
v_S the speed of the source.



Worked Example 157: The Doppler Effect for Sound

Question: The siren of an ambulance has a frequency of 700 Hz. You are standing on the pavement. If the ambulance drives past you at a speed of 20 m·s^{-1}, what frequency will you hear, when

a) the ambulance is approaching you

b) the ambulance is driving away from you

Take the speed of sound to be 340 m·s^{-1}.
Answer
Step 1 : Determine how to appoach the problem based on what is given

$$f_L = \frac{v \pm v_L}{v \mp v_S} f_S$$

$$\begin{aligned} f_s &= 700\text{Hz} \\ v &= 340\,\text{m}\cdot\text{s}^{-1} \\ v_L &= 0 \\ v_S &= -20\,\text{m}\cdot\text{s}^{-1}\,\text{for (a) and} \\ v_S &= 20\,\text{m}\cdot\text{s}^{-1}\,\text{for (b)} \end{aligned}$$

Step 2 : Determine f_L when ambulance is appoaching

$$\begin{aligned} f_L &= \frac{340+0}{340-20}(700) \\ &= 743{,}75\text{Hz} \end{aligned}$$

Step 3 : Determine f_L when ambulance has passed

$$\begin{aligned} f_L &= \frac{340+0}{340+20}(700) \\ &= 661{,}11\text{Hz} \end{aligned}$$

Worked Example 158: The Doppler Effect for Sound 2

Question: What is the frequency heard by a person driving at 15 m·s^{-1} toward a factory whistle that is blowing at a frequency of 800 Hz. Assume that the speed of sound is 340 m·s^{-1}.

Answer

Step 1 : Determine how to approach the problem based on what is given

We can use

$$f_L = \frac{v \pm v_L}{v \mp v_S} f_S$$

with:

$$\begin{aligned} v &= 340{,}6\,\text{m}\cdot\text{s}^{-1} \\ v_L &= +15\,\text{m}\cdot\text{s}^{-1} \\ v_S &= 0\,\text{m}\cdot\text{s}^{-1} \\ f_S &= 800\ \text{Hz} \\ f_L &= ? \end{aligned}$$

The listener is moving towards the source, so v_L is positive.

Step 2 : Calculate the frequency

$$\begin{aligned} f_L &= \frac{v \pm v_L}{v \mp v_S} f_S \\ &= \frac{340{,}6\,\text{m}\cdot\text{s}^{-1} + 15\,\text{m}\cdot\text{s}^{-1}}{340{,}6\,\text{m}\cdot\text{s}^{-1} + 0\,\text{m}\cdot\text{s}^{-1}}(800\ \text{Hz}) \\ &= 835\ \text{Hz} \end{aligned}$$

Step 3 : Write the final answer

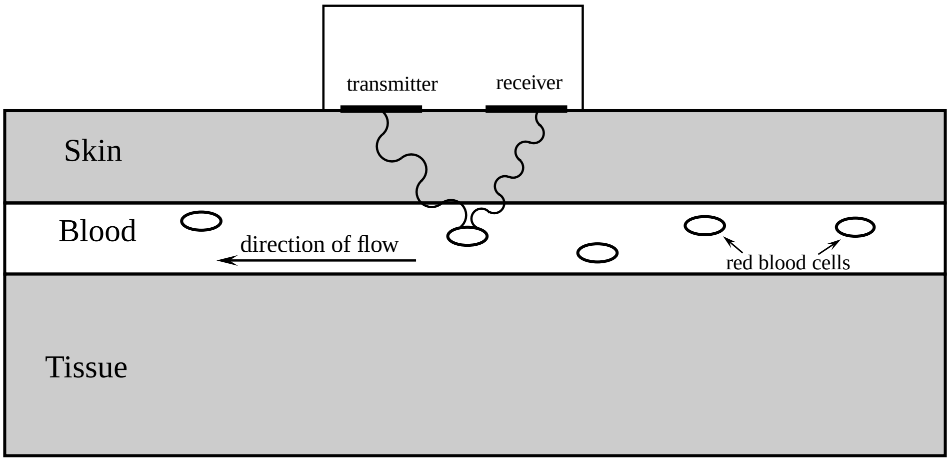
The driver hears a frequency of 835 Hz.

Interesting Fact

Radar-based speed-traps use the Doppler Effect. The radar gun emits radio waves of a specific frequency. When the car is standing still, the waves reflected waves are the same frequency as the waves emitted by the radar gun. When the car is moving the Doppler frequency shift can be used to determine the speed of the car.

24.2.1 Ultrasound and the Doppler Effect

Ultrasonic waves (ultrasound) are sound waves with a frequency greater than 20 000 Hz (the upper limit of hearing). These waves can be used in medicine to determine the direction of blood flow. The device, called a Doppler flow meter, sends out sound waves. The sound waves can travle through skin and tissue and will be reflected by moving objects in the body (like blood). The reflected waves return to the flow meter where its frequency (received frequency) is compared to the transmitted frequency. Because of the Doppler effect, blood that is moving towards the flow meter will change the sound to a higher frequency (blue shift) and blood that is moving away from the flow meter will cause a lower frequency (red shift).



Ultrasound can be used to determine whether blood is flowing in the right direction in the circulation system of unborn babies, or identify areas in the body where blood flow is restricted due to narrow veins. The use of ultrasound equipment in medicine is called sonography or ultrasonography.

?

Exercise: The Doppler Effect with Sound

1. Suppose a train is approaching you as you stand on the platform at the station. As the train approaches the station, it slows down. All the while, the engineer is sounding the hooter at a constant frequency of 400 Hz. Describe the pitch and the changes in pitch that you hear.
2. Passengers on a train hear its whistle at a frequency of 740 Hz. Anja is standing next to the train tracks. What frequency does Anja hear as the train moves directly toward her at a speed of 25 m·s^{-1}?
3. A small plane is taxiing directly away from you down a runway. The noise of the engine, as the pilot hears it, has a frequency 1,15 times the frequency that you hear. What is the speed of the plane?
4. A Doppler flow meter detected a blue shift in frequency while determining the direction of blood flow. What does a "blue shift" mean and how does it take place?

24.3 The Doppler Effect with Light

Light is a wave and earlier you learnt how you can study the properties of one wave and apply the same ideas to another wave. The same applies to sound and light. We know the Doppler effect affects sound waves when the source is moving. Therefore, if we apply the Doppler effect to light, the frequency of the emitted light should change when the source of the light is moving relative to the observer.

When the frequency of a sound wave changes, the sound you hear changes. When the frequency of light changes, the colour you would see changes.
This means that the Doppler effect can be observed by a change in sound (for sound waves) and a change in colour (for light waves). Keep in mind that there are sounds that we cannot hear (for example ultrasound) and light that we cannot see (for example ultraviolet light).

We can apply all the ideas that we learnt about the Doppler effect to light. When talking about light we use slightly different names to describe what happens. If you look at the colour spectrum (more details Chapter 30) then you will see that blue light has shorter wavelengths than red light. If you are in the middle of the visible colours then longer wavelengths are more red and shorter wavelengths are more blue. So we call shifts towards longer wavelengths "red-shifts" and shifts towards shorter wavelengths "blue-shifts".

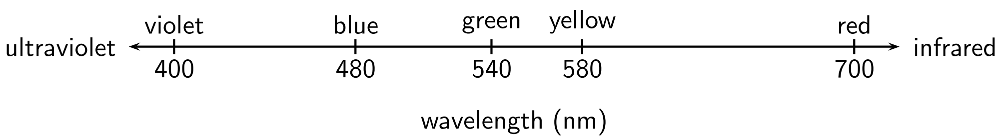


Figure 24.2: Blue light has shorter wavelengths than red light.

A shift in wavelength is the same as a shift in frequency. Longer wavelengths of light have lower frequencies and shorter wavelengths have higher frequencies. From the Doppler effect we know that when things move towards you any waves they emit that you measure are shifted to shorter wavelengths (blueshifted). If things move away from you, the shift is to longer wavelengths (redshifted).

24.3.1 The Expanding Universe

Stars emit light, which is why we can see them at night. Galaxies are huge collections of stars. An example is our own Galaxy, the Milky Way, of which our sun is only one of the millions of stars! Using large telescopes like the Southern African Large Telescope (SALT) in the Karoo, astronomers can measure the light from distant galaxies. The spectrum of light (see Chapter **??**) can tell us what elements are in the stars in the galaxies because each element emits/absorbs light at particular wavelengths (called spectral lines). If these lines are observed to be shifted from their usual wavelengths to shorter wavelengths, then the light from the galaxy is said to be *blueshifted*. If the spectral lines are shifted to longer wavelengths, then the light from the galaxy is said to be *redshifted*. If we think of the blueshift and redshift in Doppler effect terms, then a blueshifted galaxy would appear to be moving *towards* us (the observers) and a redshifted galaxy would appear to be moving *away* from us.

Important:

- If the light source is moving away from the observer (positive velocity) then the observed frequency is lower and the observed wavelength is greater (redshifted).
- If the source is moving towards (negative velocity) the observer, the observed frequency is higher and the wavelength is shorter (blueshifted).

Edwin Hubble (20 November 1889 - 28 September 1953) measured the Doppler shift of a large sample of galaxies. He found that the light from distant galaxies is *redshifted* and he discovered that there is a proportionality relationship between the *redshift* and the *distance* to the galaxy. Galaxies that are further away always appear more redshifted than nearby galaxies. Remember that a redshift in Doppler terms means a velocity of the light source *away* from the observer. So why do all distant galaxies appear to be moving away from our Galaxy?

The reason is that the universe is expanding! The galaxies are not actually moving themselves, rather the *space* between them is expanding!

24.4 Summary

1. The Doppler Effect is the apparent change in frequency and wavelength of a wave when the observer and source of the wave move relative to each other.
2. The following equation can be used to calculate the frequency of the wave according to the observer or listener:
$$f_L = \frac{v \pm v_L}{v \mp v_S} f_S$$
3. If the direction of the wave from the listener to the source is chosen as positive, the velocities have the following signs.

Source moves towards listener	v_S : negative
Source moves away from listener	v_S : positive
Listener moves towards source	v_L : positive
Listener moves away from source	v_L : negative

4. The Doppler Effect can be observed in all types of waves, including ultrasound, light and radiowaves.
5. Sonography makes use of ultrasound and the Doppler Effect to determine the direction of blood flow.
6. Light is emitted by stars. Due to the Doppler Effect, the frequency of this light decreases and the starts appear red. This is called a red shift and means that the stars are moving away from the Earth. This means that the Universe is expanding.

24.5 End of Chapter Exercises

1. Write a definition for each of the following terms.
   A Doppler Effect
   B Red-shift
   C Ultrasound
2. Explain how the Doppler Effect is used to determine the direction of blood flow in veins.
3. The hooter of an appoaching taxi has a frequency of 500 Hz. If the taxi is travelling at 30 m·s^{-1}and the speed of sound is 300 m·s^{-1}, calculate the frequency of sound that you hear when
   A the taxi is approaching you.
   B the taxi passed you and is driving away.
4. A truck approaches you at an unknown speed. The sound of the trucks engine has a frequency of 210 Hz, however you hear a frequency of 220 Hz. The speed of sound is 340 m·s^{-1}.
   A Calculate the speed of the truck.
   B How will the sound change as the truck passes you? Explain this phenomenon in terms of the wavelength and frequency of the sound.
5. A police car is driving towards a fleeing suspect. The frequency of the police car's siren is 400 Hz at $\frac{v}{35}$, where v is the speed of sound. The suspect is running away at $\frac{v}{68}$. What frequency does the suspect hear?
6. A Why are ultrasound waves used in sonography and not sound waves?
   B Explain how the Doppler effect is used to determine the direction of flow of blood in veins.

Appendix A

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